Complex Systems Modelling for Statistical Forecasting of Winter North Atlantic Atmospheric Variability: a New Approach

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Complex Systems Modelling for Statistical Forecasting of Winter North Atlantic

Atmospheric Variability: a New Approach

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1   **Abstract**

2   Seasonal forecasts of winter North Atlantic atmospheric variability have until recently shown

3   little skill. Here we present a new technique for developing both linear and non-linear statistical

4   forecasts of the winter North Atlantic Oscillation (NAO) based on complex systems modelling,

5   which has been widely used in a range of fields, but generally not in climate research. Our

6   polynomial NARMAX models demonstrate considerable skill in out-of-sample forecasts and

7   their performance is superior to that of linear models, albeit with small sample sizes. Predictors

8   can be readily identified and this has the potential to inform the next generation of dynamical

9   models and models allow for the incorporation of non-linearities in interactions between

10 predictors and atmospheric variability. In general there is more skill in forecasts developed

11 over a shorter training period from 1980 compared with an equivalent forecast using training

12 data from 1956. This latter point may relate to decreased inherent predictability in the period

13 1955-1980, a wider range of available predictors since 1980 and/or reduced data quality in the

14 earlier period and is consistent with previously identified decadal variability of the NAO. A

15 number of predictors such as sea-level pressure over the Barents Sea, and a clear tropical signal

16 are commonly selected by both linear and polynomial NARMAX models. Tropical signals are

17 modulated by higher latitude boundary conditions. Both approaches can be extended to

18 developing probabilistic forecasts and to other seasons and indices of atmospheric variability

19 such as the East Atlantic pattern and jet stream metrics.

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30   **1. Introduction**

31   Winter North Atlantic (NA) atmospheric variability is dominated by the North Atlantic

32   Oscillation (NAO; Hurrell and Deser, 2009; Hanna and Cropper, 2017). The NAO index gives

33   a measure of the pressure difference between semi-permanent high pressure over the Azores

34   and a semi-permanent low pressure over Iceland. There is a see-saw of atmospheric mass

35   between these two nodes. The greater (smaller) the pressure difference, the more positive

36   (negative) the NAO index. A positive NAO is associated with mild, wet and often stormy

37   winters over northwestern Europe while a negative NAO is linked with cold, dry conditions in

38   this region, but with wetter weather in the Mediterranean (Xoplaki et al., 2004).

39

40   The NAO can be regarded as arising from storm-track and jet-stream variability (Vallis and

41   Gerber, 2008; Stendel et al., 2016), and is an indicator of the zonality of the atmospheric flow.

42   A positive winter NAO tends to arise when the tropospheric jet and storm track are shifted

43   further northwards, driving storms towards western Europe, with a more zonal jet stream,

44   whereas a negative NAO indicates a southerly displacement with an increased meridonal jet-

45   stream component (Stendel et al., 2016) which can steer storms towards the Mediterranean and

46   enable cold air outbreaks from the Arctic to lower latitudes. The NAO is more closely

47   associated with shifts in jet latitude than it is with jet speed variability (Woollings et al., 2010a).

48

49   The NAO is a mode of internal atmospheric variability in idealised modelling experiments (e.g.

50   James and James, 1989) and until recently it was considered that NAO variability on

51   intraseasonal and interannual timescales was a result of internal atmospheric variability, or

52   climate noise (e.g. Feldstein 2000) and largely unpredictable (e.g. Johansson, 2007; Kim et al.,

53   2012). However, there is recent evidence that the storm track and jet stream are subject to

54   forcing from slowly varying boundary conditions such as ocean temperatures and sea-ice

55   changes, together with solar variability and influences from the winter stratosphere (for a

56   review see Hall et al., 2015).

57

58   A number of recent studies indicates significant potential for winter seasonal forecasting in the

59   NA region based on the influence of slowly-varying boundary conditions (Scaife et al., 2014;

60   Riddle et al., 2013; Kang et al., 2014; Dunstone et al., 2016; Stockdale et al., 2015). There are

61   also a number of older studies which identify significant skill in seasonal forecasting of the

62   winter NAO using climate models (Palmer et al., 2004; Müller et al., 2005; Derome et al.,

63   2005) and empirical approaches (Fletcher and Saunders, 2006). Scaife et al. (2014) report a

64   correlation skill of 0.62 for hindcasts of the winter NAO over the period 1993-2012, based on

65   the UK Met Office GloSea5 seasonal forecasting system (MacLachlan et al., 2015). Statistical

66   forecasts are quick and cheap to implement (e.g. Cohen et al., 2018) and therefore complement

67   the dynamical forecasts. They allow for identification of sources of potential predictability and

68   may help to explain particular instances of poor forecasts in dynamical NWP models and

69   inform their future development. Recent studies (Dunstone et al., 2016; Wang et al. 2017, Hall

70   et al., 2017) have shown promising skill in predicting the winter NAO using a linear statistical

71   framework and Folland et al. (2012) produced skillful forecasts of winter European

72   temperatures based on a number of these factors. However, these studies only use linear

73   combinations of predictors, not considering non-linear, cross-product and interaction terms.

74   Often statistical models can be constructed for a training period with a very good fit,

75   subsequently failing when making out-of-sample forecasting, due to non-stationary

76   relationships, internal variability and overfitting in the training period.

77

78   The slowly varying boundary conditions may act to reinforce or oppose one another and

79   numerous studies examine remote causes of NA atmospheric variability, mostly in a linear

80 framework. However, the interaction of the atmosphere with boundary forcing can be non-

81 linear (Petoukhov and Semenov, 2010), so a purely linear approach may only capture a limited

82 portion of the variability. Here we further develop statistical seasonal forecasting by using a

83 novel application of NARMAX (Non-linear Auto-Regressive Moving Average with

84 eXogenous inputs) methodology (e.g. Billings, 2013), comparing linear and polynomial

85 regression-based forecasting models. We aim to investigate whether the inclusion of non-linear

86 interactions help to explain changes in the NAO. NARMAX modelling can reveal and

87 characterize non-linear dynamic relationships among signals from recorded data and produces

88 transparent models which demonstrate how a response variable (system output signal) is linked

89 to a number of candidate explanatory variables (system input signals) and their combined

90 interactions. The NARMAX approach will construct the simplest model to explain the system:

91 therefore if a linear model provides a good representation of the system, the NARMAX method

92 will go no further (Billings, 2013, p9). NARMAX modelling was first introduced to solve non-

93 linear dynamical system identification and modelling problems in engineering, and it has been

94 successful in revealing linear and non-linear relationships at a wide range of scales within the

95 engineering, biological, ecological, medical, geophysical, and environmental sciences (e.g.

96 Billings, 2013; Bigg et al., 2014; Ayala-Solares et al., 2018).

97

98 Here we review some of the drivers of North Atlantic climate variability identified in previous

99 research. A number of studies identify a sea-ice influence, particularly from the Barents-Kara

100 Sea (BK) region (e.g. Koenigk et al., 2016; Garcia-Serrano et al., 2017). A likely pathway of

101 influence is due to constructive interference of the atmospheric warming related to autumn sea-

102 ice loss with climatological planetary wave patterns (Screen et al., 2018, Wu and Smith, 2016).

103 The location of the BK region is close to the climatological ridge of the zonal wave-1 and

104 wave-2 planetary waves, with localised warming acting to reinforce this pattern (Zhang et al.,

105 2018) which enhances vertical wave propagation. This then can weaken the stratospheric polar

106 vortex with a subsequent downward propagation of this signal over a number of weeks

107 (Baldwin and Dunkerton, 2001). A stronger (weaker) vortex is associated with a positive

108 (negative) NAO, as a strengthened (weakened) circumpolar stratospheric jet induces a

109 poleward (equatorward) shift in the tropospheric jet and storm tracks (Kidston et al., 2015). A

110 cryospheric influence has also been detected from Siberian snow anomalies, which may

111 enhance the Siberian high-pressure region, resulting in vertical wave propagation into the

112 stratosphere and a weakening of the stratospheric vortex (Cohen et al., 2007). This effect has

113 been observed in models, but can be weak, and may require modulation by the Quasi-biennial

114 Oscillation (QBO) to be more effective (e.g. Tyrrell et al. 2018). The strength of the vortex can

115 also be perturbed by factors such as the QBO phase (Boer and Hamilton 2008), the solar cycle

116 (Ineson et al., 2011) and tropical volcanic eruptions (Robock and Mao,1995; Driscoll et al.,

117 2012).

118

119 Sea-surface temperatures (SST) are an important element of boundary layer forcing. The

120 interaction between atmosphere and ocean is complex, with the atmospheric variability

121 characterised by the NAO forcing SST variability in the NA, to produce the distinctive tripole

122 pattern of SSTs (Deser et al., 2010). However, there is evidence for feedback of this SST

123 pattern to the atmosphere at time lags of a few months, as the spring tripole anomalies are

124 preserved beneath the summer mixed layer, re-emerging in winter as the mixed layer deepens

125 (Rodwell et al., 1999, Deser et al., 2003; Czaja and Frankignoul, 1999). A complementary SST

126 pattern is the North Atlantic Horseshoe (NAH; Czaja and Frankignoul, 2002), which may

127 evolve from the tripole anomalies and where SST anomalies may lead the NAO by up to six

128 months. These patterns fluctuate on a decadal scale, but Atlantic SSTs also experience

129   multidecadal variability known as the Atlantic Multidecadal Oscillation (AMO, e.g. Enfield et

130   al., 2001) with warm and cool phases and a period of 65-80 years.

131

132   Other recently identified associations involve SSTs in the BK and Greenland-Norwegian (GIN)

133   Seas (Kolstad and Årthun, 2018). In addition, sea-level pressure (SLP) in the BK region can

134   precondition autumn sea-ice extent, and so itself may be a potential predictor of European

135   winter weather variability (King and Garcia-Serrano, 2016). Furthermore, the temperature

136   variability of the NA subpolar gyre (SPG) has been associated with changes in jet speed. A

137   weakened gyre circulation can lead to increased poleward transport of warmer subtropical

138   waters, and a decrease in meridional temperature gradient (Häkkinen et al., 2011; Woollings

139   et al., 2018).

140

141   The role of solar fluctuations in North Atlantic climate variability is a subject of considerable

142   debate. A solar cycle signal has been detected in European winters (Lockwood et al., 2010;

143   Woollings et al., 2010b), with lower solar activity associated with colder European winters.

144   Recent studies suggest that there is a lag of 3-5 years between the solar signal and its impact

145   on the atmosphere, measured by the NAO (Scaife et al., 2013; Gray et al., 2013; 2016; Andrews

146   et al., 2015), possibly as a result of integration of the solar signal over time by SSTs.

147

148   The influence of tropical teleconnections is also evident. The El-Niño-Southern Oscillation

149   (ENSO) signal can propagate via troposphere and stratosphere (Toniazzo and Scaife, 2006;

150   Bell et al., 2009) and there is evidence of non-linearity, with stronger El Niño events not having

151   the NAO-like impact of moderate events (Folland et al., 2012; Rao and Ren, 2016a; 2016b).

152   This may be a consequence of a more eastward centre of action for stronger events (Takahashi

153   and Dewitte, 2015) with a different pathway of propagation. In addition, links between Indian

154   Ocean SST anomalies and the NAO, (Hoerling et al., 2004; Li et al., 2010) with the NAO

155   lagging SST anomalies by a month or more and between the NAO and the Madden-Julian

156   Oscillation (Garfinkel et al., 2014 Tseng et al., 2018) have also been identified. Yu and Lin

157   (2016) find an NAO response to tropical heating anomalies in the Indian and Atlantic regions,

158   although not necessarily to SSTs.

159

160   Influences from these boundary conditions at a range of lead times means there could be a

161   significant component of predictability within the winter NA atmospheric circulation (Smith

162   et al., 2016). As the NAO is such a significant factor in determining the winter weather around

163   the Atlantic basin, skillful seasonal forecasts of the NAO will have considerable economic and

164   societal benefits. The NAO is related to hydrological outlooks and flood risk (e.g. Svensson et

165   al., 2015; Bell et al., 2017) and is significantly related to energy demand (Thornton et al., 2017;

166   Clark et al., 2017).

167

168   Section 2 presents the data used, and section 3 explains the methods, including how the

169   NARMAX methodology is applied. Results are presented in section 4 and interpreted in section

170   5. This is followed by some concluding remarks in section 6.

171

2. Data

173   Data used are summarised in Table I with additional information in Table S1. HadISST1

174   (Rayner et al., 2003) is used for SST-based and sea-ice variables. We select monthly predictor

175   variables taken at lead times of one month up to a maximum of seven months (the preceding

176   May) preceding the winter in question. For this study it is assumed that there is no prediction

177   skill derived from the previous year's NAO, although November values are available for

178   selection. The sea-ice regions are taken from Screen (2017), where nine distinct sectors are

179 identified with limited covariability suggesting a large degree of regional independence. The

180 NA SST tripole index is constructed following Fan and Schneider (2012). Similarly, the NAH

181 and SST gradient indices are derived by calculating mean SSTs over two regions and

182 subtracting them. See Table S1 for details. Blanca Ayarzaguena provided the T100 index which

183 is a measure of the strength of the stratospheric polar vortex (SPV). This is an index of daily

184 temperature anomalies at 100hPa, averaged over 65-90N, derived from JRA-55 reanalysis data

185 (Kobayashi et al., 2015). Monthly mean values are constructed from an average of daily means.

186 Tropical rainfall anomalies provide an indication of convective activity and divergence aloft,

187 which can generate Rossby waves which propagate away from the source and are capable of

188 influencing extratropical atmospheric circulation (Hoskins and Karoly, 1981). Data are

189 obtained from the Global Precipitation Climatology Project (GPCPv2.3, Adler et al., 2003).

190 However, such data are not available prior to 1979 so tropical SSTs for similar regions are used

191 in the 1956 models, although there is not always a causal link between SSTs and tropical

192 rainfall, depending on the region under consideration. The MJO is another index which can

193 provide a tropical signal and is obtained from the Climate Prediction Center (CPC). Here ten

194 phases are used rather than the more common eight, and a negative value indicates the active

195 convective state (Baxter et al., 2014). QBO data (Naujokat, 1986, updated) use the 30hPa level

196 following Hamilton (1984). All data are normalised to the period 1981-2010.

197

198 To capture the non-linear relationship between the NAO and the ENSO signal, we use the

199 discontinuous N3.4 index of Folland et al. (2012) alongside the conventional index, where

200 values are set to zero for normalised values between ±1. More negative values are set to -1,

201 values in the range 1-1.75 are set to one, while values greater than 1.75 are set to zero. The

202 volcanic index used by Folland et al. (2012) is also applied. Here, for two years following a

203 volcanic eruption, values are set to one, and are zero for all other years, as Robock and Mao

204   (1995) identify the effects of eruptions in the first and second winters after the eruption. This

205   simulates the persistence of volcanic aerosols in the stratosphere after a major eruption. Major

206   tropical eruptions are as identified by Stenchikov et al. (2006).

207

208   Two versions of the NAO are used: the PC-based NAO (HPC; Hurrell et al., 2003) and a

209   station-based index derived from SLP data from Reykjavik and Ponta Delgada, Azores (station

210   NAO). This is taken as the difference between SLP at the two stations, which is then

211   normalised. This is the approach followed by Scaife et al. (2014), although other indices are

212   typically constructed by normalising the pressure at each station before subtracting (e.g.

213   Hurrell, 1995; Cropper et al., 2015). The correlation between these two indices for winter is

214   0.90 (1956-2017); however, this does vary slightly over time (13-year running correlations

215   between the two reach a low of 0.82 for a period in the early 1990s). The PC-based NAO better

216   captures the spatial patterns of the NAO, although is dependent on the area selected for

217   analysis, whereas due to shifts in the NAO centres of action a fixed-point index will not always

218   represent this optimally. A disadvantage of the PC-based approach is that the whole index time

219   series needs recalculating every time a new value is incorporated, and being a mathematical

220   construct, does not necessarily represent climate physics (Dommenget and Latif, 2002)

221

222   Data are not detrended as we aim to forecast as closely as possible to the real world. In order

223   to address any trends present in the data, indices of atmospheric carbon dioxide and global

224   temperature are available for incorporation in the models. The winter season is defined as

225   December-January-February (DJF), where the winters refer to the year of the January.

226

227   **3. Methods**

228   Models are constructed for training periods from 1956-2010 and 1980-2010. The year 2010 is

229   used as the cutoff, as in that year there are extreme values of the NAO and these are included

230   in the training dataset, as simulation experiments show that models trained without extreme

231   values perform poorly when used for prediction. Such extreme values are particularly

232   important in view of the relatively small sample sizes available. This leaves 2011-2018 for use

233   as a validation period (2011-2017 for the station-based NAO due to data availability). We

234   predict values for this period without adapting the model. An alternative, retroactive

235   verification approach (Mason and Baddour, 2008) constructs a model over the training period,

236   then forecasts the next year only based on that model. Forecasts for subsequent years are based

237   on models which incorporate the previous year's observation, and the models are allowed to

238   evolve, both in terms of coefficients and predictors selected. This approach was tested for linear

239   models and produced almost identical forecasts with only slight changes in coefficients, but

240   the same predictors, with no appreciable improvement in forecast quality.

241

3.1 The NARMAX Method

243   One of the most attractive features of the NARMAX model, distinguishing it from other non-

244   linear data-driven modelling techniques, is its power to build transparent and interpretable

245   models where the mathematical significance of each model term is meaningful (e.g. Billings,

246   2013, p10) and in most real applications the selected model terms are physically interpretable

247   (e.g. Billings, 2013, Chapter 14 Case Studies). Essentially, this approach treats each of the

248   candidate predictors as a possible underlying cause of change in the response variable of

249   interest and uses a set of model detection algorithms to automatically identify and pick out the

250   most important predictors, based on which it then establishes a quantitative relationship that

251   best relates the possible forcing variables to the response variable. Note that the dependence

252 relation of response on potential predictors can be either linear or non-linear. While traditional

253 linear modelling approaches such as ARMA and ARMAX might be able to capture main linear

254 relationships, they fail to reveal or capture any non-linear dynamics that are inherent in weather

255 and climate (e.g. Easterling et al., 2000; Hoerling et al.,2001; Dell et al., 2013; Burke et al.,

256 2015).

257

258 Taking the case of a one input (u) and one output (y) problem as an example, the NARMAX

259 model for y is written as (Billings, 2013):

$$
\begin{aligned}
y(k) = F(&y(k-1), y(k-2), \mathsf{L}\ , y(k-n_y), \\
& u(k-d), u(k-d-1), \mathsf{L}\ , u(k-d-n_u), \\
& e(k-1), e(k-2), \mathsf{L}\ , e(k-n_e)) + e(k)
\end{aligned}
\tag{1}
$$

260 where $y(k)$ and $u(k)$ are the measured system output (response) and input (explanatory),

261 respectively, at time k; $e(k)$ is a noise sequence which is not measurable but can be estimated

262 once a model is built; n_y, n_u, and n_e are the maximum lags for the system output, input, and

263 noise; $F(\bullet)$ is some non-linear function to be determined; and d is a time delay (typically d=0 or

264 d=1). For an identified model, the noise $e(k)$ can be estimated as the prediction errors:

265 $e(k) = y(k) - \hat{y}(k)$, where $\hat{y}(k)$ is the predicted value at time instant k generated by an

266 estimated model. The noise terms are included to accommodate the effects of measurement

267 noise, modelling errors, and/or unmeasured disturbances. For the purposes of this study, we

268 consider the predictability that can be obtained from the input (predictor) variables of preceding

269 months, and so do not consider past outputs (previous winter NAO values) further.

270

271 There are many model subset selection methods such as the traditional forward selection

272 (Faraway, 2002; Wilks, 2011). NARMAX uses an orthogonal forward selection algorithm,

273 called Forward Regression Orthogonal Least Squares (FROLS; Billings, 2013), to select the

274   important terms. The efficiency of FROLS can be attributed to its use of mutual information

275   (in addition to the correlation function), to measure not only linear but also potential non-linear

276   dependent correlation of the target signal and the candidate explanatory signals. Furthermore,

277   unlike traditional stepwise selection which uses hypothesis-tests and p-values to measure the

278   significance of variables (terms), FROLS uses an effective index called the error reduction

279   ratio (ERR) to measure the contribution made by each of the individual terms to explaining the

280   change in the response variable, based on which the most significant term will be selected in

281   each search step. The number of model terms can be determined using either the APRESS

282   (Billings and Wei, 2008) statistics or the penalized error-to-signal ratio (PESR, a variant of

283   APRESS) proposed in Wei et al. (2010). This is similar in principle to the Akaike Information

284   Criterion (AIC) and Bayesian Information Criterion (BIC) but is developed specifically for

285   non-linear systems. A leave-K out cross-validation is normally used with NARMAX, where K

286   is around 10% of the training sample. A common model is identified from subsets of the

287   training period, and common model parameters are then estimated using all the training data.

288   In this study, a leave-one-out approach is employed due to the small sample size in the training

289   data.

290

3.1.1. Linear models

291

292   NARMAX includes several linear model structures, e.g. autoregressive with exogenous inputs

293   (ARX) and autoregressive moving average with exogenous inputs (ARMAX) as a special case.

294   A general linear model structure of NARMAX is as follows:

295
$$y(k) = a_1 y(k-1) + \ldots + a_{n_y} y(k-n_y) + b_0 u(k-d) + b_1 u(k-d-1) \ldots + b_{n_u} u(k-n_u),$$
$$+ e(k) + c_1 e(k-1) + \ldots + c_{n_e} e(k-n_e)$$
(2)

296 where $a_1,a_2,...,a_{n_y}$, $b_0,b_1...,b_{n_u}$, and $c_1,c_2,...,c_{n_e}$ are model parameters. The above single input

297 single output (SISO) case model (2) can be extended to multiple input single output (MISO)

298 or multiple input multiple output (MIMO) cases. The commonly used multiple linear

299 regression model is a special case of the MISO linear model. For example, for a case where

300 there are r inputs, $u_1, u_2,..., u_r$, by setting $d=0$, $n_y=0$, $n_u=0$ and $n_e=0$ yields:

$$y(k) = a_1u_1(k) + a_2u_2(k) + ... + a_ru_r(k) + e(k) \qquad (3)$$

302 **3.1.2. Polynomial NARMAX models**

303 In practice, many types of model structures are available to approximate the unknown function

304 $F(\bullet)$ in (1), including power-form polynomial models and rational models (Chen and Billings,

305 1989), radial basis function networks (Wei et al., 2007), and wavelet neural networks (Billings

306 and Wei, 2005; Wei et al., 2010). Power-form polynomial models are the most commonly used

307 representation because such models have a number of unique, attractive properties (Billings

308 2013, pp35-37): for example for most applications the resulting models are transparent,

309 physically interpretable and simple (parsimonious). This is the model form used in this study.

310

311 In practical applications, it is usual to consider many input signals (or explanatory variables)

312 and investigate how the explanatory variables (e.g. $u_1, u_2,..., u_r$) influence the response variable

313 of interest. A case of many inputs can be represented by a special form of the NARMAX model

314 as follows:

$$y(k) = f(u_1(k),u_1(k-1),\text{L} ,u_1(k-n_u),\text{L} ,u_2(k),u_2(k-1),\text{L} ,u_2(k-n_u),\text{L} ,$$
$$u_r(k),u_r(k-1),\text{L} ,u_r(k-n_u)) + e(k) \qquad (4)$$

315 where $n_u \geq 0$. In this study, $n_u = 0$, for which the model reduces to a polynomial model which

316 belong to the family of multiple linear regression models:

$$y(k) = f(u_1(k),u_2(k),...u_r(k)) + e(k) \qquad (5)$$

318   For example, with two inputs u_1 and u_2, the initial full model of degree 2 is:

319    $$y(k) = a_0 + a_1 u_1(k) + a_2 u_2(k) + a_3 u_1{}^2(k) + a_4 u_1(k) u_2(k) + a_5 u_2{}^2(k) + e(k) \quad (6)$$

320   Note that in practice it may not be necessary for all the six model terms in (6) to be included in

321   a final predictive model, and only those that are important for explaining the variation of the

322   response should be included in the final model.

323

3.2. Small Sample Size Problem and Model Averaging

325   The NARMAX method has been successfully applied to solve a wide range of real-world

326   problems, and in most cases the method produces a robust reliable model that can be used for

327   system analysis and prediction (simulation). For small sample-size problems where the number

328   of observations is small and much smaller than the number of regressors (explanatory variables

329   and their cross-product interactions), the identified model can be sensitive to adding or

330   removing a variable or cross-product term. In order to reduce the risk of using a single model

331   and mitigate the uncertainty in the prediction of a single model, this study proposes a simple

332   model-averaging approach to deal with the small sample-size problem. Models used in model

333   averaging only differ from each other in the number of terms used, thus all contain a common

334   core of predictors, with only minor differences in the coefficients.

335

336   Assume the training dataset S contains a total of N data points. Rather than using only a single

337   model to calculate predictions, we use a weighted average of multiple models (in this case

338   three) to carry out predictions. The model averaging scheme is described below.

339

340   Let $M_1, M_2, ..., M_s$ be the s best models identified by the PESR from the training dataset S, and

341   assume the values of the mean squared errors (MSE) of the s models over the training dataset

342   are $mse_1, mse_2, ..., mse_s$, respectively. Define

$$c_1 = 1/mse_1, c_2 = 1/mse_2, ..., c_s = 1/mse_s \tag{7a}$$

$$c = c_1 + c_2 + ... + c_s \tag{7b}$$

$$w_1 = c_1/c, w_2 = c_2/c, ..., w_s = c_s/c \tag{7c}$$

346   Let $\hat{y}_1, \hat{y}_2, ..., \hat{y}_s$ be the predicted output values by the s models, the model averaging

347   prediction is defined as:

$$\hat{y} = w_1 \hat{y}_1 + w_2 \hat{y}_2 + ... + w_s \hat{y}_s \tag{8}$$

349

3.3. Forecast Verification

351   Correlation, Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) are used to

352   give an indication of forecast skill, for both training and testing periods. A further measure is

353   used, based on the mean squared error (MSE), the Mean Square Error Skill Score (MSESS,

354   e.g. Wilks, 2011, p328). This compares the skill of the forecast with a reference forecast, in

355   this case a forecast based on climatology. There is generally lower skill in seasonal forecasting

356   compared with shorter term forecasts and an out-of-sample forecast of 8 years is not large

357   enough to allow robust statistical conclusions to be drawn about the quality of the forecast. A

358   generalized discrimination score, D, (Mason, 2012; Mason and Weigel, 2009) indicates

359   whether the forecasts are potentially useful, despite bias and poor calibration in a low-skill

360   situation (Mason, 2012), based on whether forecast values increase (decrease) with an increase

361   (decrease) in observation values, regardless of the magnitude of the change. For a pair of

362   observations, D gives the probability that a forecaster can discriminate the observations based

363   on the corresponding forecasts. D is related to Kendall's correlation coefficient τ by:

364 $$\tau = 2D - 1 \tag{9}$$

365 τ is effectively scaled from 0 to 1, a value of 0.5 means there is no skill in the forecast as the

366 probability of correctly discriminating between the size of two observations is 50%. Values

367 greater than 0.5 suggest that the forecast is potentially useful.

368

369 Comparing the forecast methods objectively and determining whether there is a significant

370 difference is difficult with a small out-of-sample testing set (N=8; seven for station NAO). To

371 address this, the longer time series is split into even and odd years for the station NAO data.

372 Models are trained on the even years (N=32), the odd years forming the testing period (N=31).

373 The differences between linear and polynomial NARMAX model correlations for the testing

374 period are assessed for significance with the "cocor" package for R (Diedenhofen and Musch,

375 2015), using tests for overlapping dependent samples (the correlations both concern the NAO

376 and the compared coefficients have a shared variable, the observed NAO). The package runs

377 six different tests and their variations, for assessing the significance of the difference between

378 the correlations, together with a confidence interval test. For details see Diedenhofen and

379 Musch (2015).

380

381 A further test is performed to examine whether the results obtained in the testing period for

382 NARMAX models are likely to have occurred by chance. For the HPC80LIN model, 100

383 surrogate datasets (Kugiumtzis, 2000; Schrieber and Schmitz, 2000) are created for each of the

384 predictors selected, allowing 100 surrogate NAO timeseries to be constructed for each of the

385 models with different numbers of terms (models with five to nine terms were examined).

386 Further details of this approach are given in the online supplementary information.

387

388 **4. Results**

389   In this section, NAO models are designated by type (station-based NAO: stat; Hurrell principal-

390   component NAO: HPC), by year, and whether linear (LIN) or polynomial (POLY) NARMAX

391   models. Thus stat80POLY is the polynomial NARMAX model for the station-based NAO,

392   using 1980-2010 as the training dataset.

393

4.1 Linear Models

395   The predictors used in the linear models, and their coefficients are summarised in Table II and

396   verification statistics for the averaged models are shown in Table III. It is the averaged models

397   that are subsequently discussed, unless otherwise stated. An example of the model fit is shown

398   for the station-based NAO in Figure 1, with other models shown in Figure S2. There is some

399   consistency among the predictors selected for the different NAO indices, both within and

400   between the different training periods (Table II). October Barents Sea SLP and October

401   Barents-Kara Sea ice are selected for all models. Bering Sea and East Siberian-Laptev Sea ice

402   in November are selected for three of the four models; both 1980 linear models, and HPC56LIN

403   (Bering Sea ice only) and stat56LIN (East Siberian-Laptev Sea ice only). There is some

404   discrepancy in the cryospheric terms, with Greenland Sea ice being selected only for the 1956

405   models. While Greenland Sea ice is chosen in the 1956 NAO models, October Bering Sea ice

406   is selected for HPC80LIN as an additional term. Tropical influences are identified in both 1980

407   models, and for stat56LIN, however these show considerable variation, as variables selected

408   for the 1980 models such as the MJO and tropical rainfall, are not available for the 1956

409   models. Consequently the greater availability of tropical predictors post-1980 is reflected in

410   these models: November MJO (phase 8, HPC80LIN and phase 9, stat80LIN) and tropical

411   Atlantic rainfall (July; stat80LIN) and August (HPC80LIN) are present in both 1980 linear

412   models. Despite being previously identified as an important predictor of winter extratropical

413   atmospheric variability, N3.4 is not particularly prominent, only being selected for both station-

414   based NAO models (July N3.4I). Extratropical oceanic SST-related terms are selected for all

415   except stat80LIN; October GIN SST for HPC80LIN and NAH (September-stat56LIN,

416   October-HPC56LIN). The only model to detect a stratospheric influence is HPC56LIN, where

417   the stratospheric polar vortex indices for October and November are selected, although

418   interestingly these are of opposite signs and are only weakly correlated (r=0.21).

419

420   Correlations of the 1980 models for the 2011-2018 testing period (Table IIIa) are 0.90

421   (HPC80LIN) and 0.82 (stat80LIN). However, correlations can mask a systematic negative bias

422   in the forecasts, particularly in the linear model forecasts (with the exception of HPC80LIN),

423   which becomes evident when MAE and RMSE are examined. For example, although

424   stat80LIN correlates very well in the testing period (r=0.82), MAE and RMSE are 1.11 and

425   1.26 respectively, considerably higher than the training period values. MSESS scores are

426   negative, the forecasts having less skill than a climatological forecast. The correlations indicate

427   that the two NAO models are able to capture aspects of the local maxima and minima of the

428   observed NAO during the testing period, but with a large negative bias (Figure 1), which

429   reduces skill according to MSESS and inflates the MAE and RMSE. However, D-scores

430   suggest that the models for the NAO have considerable potential usefulness (0.80, stat80LIN;

431   0.75, HPC80LIN, Table IIIa).

432

433   For the 1956 linear models, over the testing period the correlation is weaker for both

434   (HPC56LIN, 0.46; stat56LIN, 0.69; Table III) compared with the 1980 models although no

435   significant difference can be determined due to small sample size. MAE and RMSE values for

436   test data are similar to the 1980 linear model for the station NAO, but greater for the HPC

437   NAO, but are always larger than those for the training period, reflecting the negative bias of

438   forecasts in the testing period. MSESS are negative, but D-scores indicate potential usefulness

439   (0.80, stat56LIN; 0.68, HPC56LIN).

440

441   Many of the terms used in construction of the linear models, for example sea-ice, SST and

442   tropical rainfall terms contain significant trends, which are likely to contribute to the negative

443   bias of the forecasts. It is notable that the station NAO forecasts from 1980 training data appear

444   to have overall negative trends (Figure 1), probably due to the influence of the sea-ice inputs

445   to the models, as found in Hall et al. (2017).

446

447   **4.2 Polynomial Models**

448   The polynomial models are presented in detail in Table IV and compared with linear models

449   in Figures 1 and S2. The comparisons use the averaged linear and polynomial models and an

450   example of model averaging for the stat80POLY model is shown in Figure S3. Verification

451   statistics are in Table III. For the HPC80 forecast the NARMAX algorithm produces a linear

452   model only, as the inclusion of interaction terms does not significantly improve prediction

453   performance.

454

455   There is less consistency amongst the predictor variables selected (Table IV) compared with

456   the linear models, as terms are more complex and often based on interactions between

457   variables, and with short datasets the models are very sensitive to slight differences in

458   predictors. Equally, different predictors selected may be trying to capture the same variability

459   or acting as proxies for some hidden variable. Some predictors are selected for both linear and

460   polynomial models but may differ by a month or two.

461

462   Forecasting verification statistics for the polynomial model averages are shown in Table III. In

463   almost all cases, MAE, RMSE, MSESS and correlation are superior to the linear models in

464   both training and testing periods, for both time periods. The D-score is consistently greater than

465   0.7, indicating considerable potential usefulness of the forecasts and MSESS are always

466   positive. Verification statistics are mostly poorer for the 1956 models. Initially this may appear

467   to be surprising as the models are constructed over a longer training set, although data from the

468   earlier part of the time series prior to the satellite era are of poorer quality, and some variables

469   are not available for the earlier period. See also section 5.1 on variable inherent predictability

470   over the time period. The 1956 polynomial models show considerable improvements in all

471   verification metrics compared with linear models on the longer time-scale, although results

472   from linear models are broadly comparable to results obtained using the shorter training set.

473   Due to the small sample size, the statistical significance of any improvements achieved by

474   polynomial over linear models cannot always be satisfactorily determined. As would be

475   expected for such a small sample size, when correlations in the testing period are compared

476   using the suite of tests in the "cocor" package, differences are not significant.

477

478   In order to create a longer testing period and provide greater confidence in the performance of

479   the models in out-of-sample prediction, the 1956-2017 period was split into even years for the

480   training dataset and odd years for the testing dataset, using the station-based NAO index. The

481   predictive skill of NARMAX polynomial and linear models when even years only are used as

482   the training period is shown in Figure S4 and verification statistics are presented in Table S2.

483   Correlations for the testing period (odd years) are 0.45 (linear) and 0.60 (polynomial). While

484   both are significant ($p<0.05$), the polynomial correlation coefficient is appreciably higher,

485   consistent with results from the shorter testing datasets, yet still not deemed significantly so by

486   the range of comparison tests in "cocor". However, this does not tell the whole story. For the

487 earlier part of the testing period (1957-1979), correlations for the linear and polynomial models

488 are very similar (0.6, linear; 0.66, polynomial), however in the later period (1981-2017), the

489 correlations are significantly different using "cocor" (0.04, linear; 0.44 polynomial, p<0.1, one-

490 tailed test). In addition both MAE and RMSE for 1981-2017 are around two thirds of the values

491 for 1957-1979. This is suggestive of some change in the NAO in the later period, which is

492 better captured by the polynomial models.

493

494 Verification statistics taken over the 100 surrogate models for HPC80LIN show that model

495 performance is much poorer for the surrogates in the testing period (Table S3), with significant

496 differences between model performance and the surrogate data for all verification metrics. This

497 indicates that the good prediction performance obtained with NARMAX is not a result of

498 chance but is due to the efficacy of the algorithm.

499

500 Figure 2 allows closer inspection of model performance in the testing period. Both linear and

501 polynomial 1980 models capture the local maxima and minima of both versions of the NAO

502 for the first three years (Figure 2a,b), although the amplitude of the response is much reduced

503 in the stat80LIN model (Figure 2b), which also display a negative bias, as discussed above,

504 while any bias in stat80POLY is minimal and non-systematic. Similarly1956 linear models for

505 both NAO indices reproduce the interannual change from 2011-2014, after which the response

506 is damped. In general, polynomial models better match the local maxima and minima, with

507 the exception of HPC56POLY which forecasts a negative rather than positive NAO in 2014

508 (Figure 2c).

509

510 Figures 3 and 4 give a visual representation of the selected predictors for the different NAO

511 models, for both training periods, in order to highlight common predictor variables, although

512   some of these are used in interaction terms in the NARMAX models. For 1980 (Figure 3),

513   Barents Sea SLP and Bering Sea ice are used in all three models, Barents-Kara Sea ice, East

514   Siberian-Laptev Sea ice and tropical Atlantic rainfall are selected by both linear models while

515   MJO phase 8 occurs for both linear and polynomial HPC models. For the 1956 models (Figure

516   4), some predictors (Greenland Sea ice, NAH, Barents Sea SLP) are included in all four models,

517   while October Barents-Kara Sea ice and the N3.4I index are selected in three. A number of

518   predictors occur in only one model, due to them occurring as interaction terms, and the

519   sensitivity of the model to slight changes in variables given the short time series. For the 1956

520   training data, linear and polynomial models will select predictors that were suboptimal in the

521   1980 models, as the best predictors are not always available for the longer time series. Thus

522   NARMAX selects N3.4I for the 1956 models, whereas for the 1980 models the optimal tropical

523   influences seems to come from the MJO and tropical rainfall. While models are capturing

524   essentially the same signals, there is high sensitivity to small variations in input data due to

525   small sample size: hence different predictors are selected, which represent slightly different

526   aspects of a common signal.

527

528   From the results, it is remarkable that while linear models show more limited skill, particularly

529   over the longer training dataset, polynomial model forecasts, whether linear or non-linear,

530   particularly those based on the 1980 training set, are able to replicate local maxima and minima,

531   the amplitude of the observation and have minimal bias for the testing dataset.

532

5. Discussion

5.1 Differences when using longer and shorter training periods

535   Based on the verification statistics (Table III), polynomial models generally outperform the

536   equivalent linear model (lower MAE and RMSE; higher correlation, MSESS and D-score),

537 although the significance of this is hard to assess on a small sample size. 1956 models generally

538 perform less well than the equivalent from 1980. There is evidence from other studies to

539 suggest that from the 1950s to 1960s there is less skill in dynamical forecasting hindcasts, with

540 the possibility that during this period the atmospheric circulation was inherently less

541 predictable (e.g. Weisheimer et al., 2016), associated with a more prevalent negative NAO.

542 However, dynamical models appear to be skillful in predicting strongly negative NAO events,

543 but less skillful regarding weaker negative NAO events. O'Reilly et al. (2017) concur that in

544 the mid 20th century, forecast skill for the NAO is reduced, and attribute this to weaker forcing

545 from tropical Pacific SSTs during this period. An alternative perspective is presented by

546 Woollings et al. (2015). Decadal variability of the NAO is identified, which is associated with

547 changes in strength of the jet and different dynamical behaviour (eddy-mean flow interaction,

548 Rossby wavebreaking and blocking), the period from 1980 being associated with relatively

549 higher jet speeds. Therefore when statistical models are trained on a longer period, with the

550 assumption of stationarity, these decadal variations are averaged out and predictors that are

551 selected are likely to be sub-optimal. Furthermore, if there is a return to relatively weak forcing

552 from Pacific SSTs, or a period of reduced jet speeds, forecasts trained on the recent period may

553 perform less well (O'Reilly et al., 2017). Weisheimer et al. (2016) report that forecast skill

554 actually increases further back in the 20th century, in the 1930s and 1940s, suggesting the issue

555 may not be attributed to reduced data quality. This corresponds to the decadal fluctuations

556 identified by Woollings et al. (2015), suggesting that periods of lower jet speed may contribute

557 to the reduced predictability. In this study, a longer training dataset does not equate to a better

558 forecasting model, in agreement with the studies above. It is also notable that when tested on

559 all the odd years, a polynomial model significantly outperforms a linear model for the period

560 1981-2017, better capturing the NAO for both early and late periods (Figure S4, Table S2).

561

562   Greenland Sea ice is a predictor in all 1956 models (Figure 4) but is not selected for models

563   based on the shorter training period. The months selected (May-July) represent the time of

564   maximum decrease in the regional annual sea-ice cycle (October is selected as the 8th and final

565   term in stat56LIN, Table IIc, and makes a minimal contribution). Interannual variability and

566   mean values of Greenland Sea ice are notably larger prior to 1980; consequently contributions

567   of the predictor terms to the models are greatest prior to 1980. Given the relatively small

568   coefficients used in the models, contributions between 1980 and 2010 are negligible, also

569   indicating why this variable is not selected for the 1980 models. Figure S5 shows that the four

570   Greenland Sea ice terms used essentially make similar contributions to each model, and can be

571   regarded as different, somewhat imperfect representations of some unknown predictor variable.

572   It is possible that the association in the 1960s and 1970s is related to the Great Salinity Anomaly

573   (e.g. Dickson et al., 1988), which circulated in the North Atlantic at this time and had its origins

574   in increased ice export and freshwater release through the Greenland Sea region. The

575   association between the Greenland Sea ice and NAO is physically plausible at this time,

576   particularly as the predictors capture the transition from a more negative to a positive NAO

577   over the period.

578

579   **5.2. Tropical forcing and interaction terms**

580   Evidence for forcing of the winter NAO from the tropics is evident in all models; the 1980

581   training data favours the selection of the MJO and tropical rainfall, whereas these are not

582   available for the 1956 training data, so the N3.4 Index becomes more prominent, along with

583   West Indian Ocean SST in stat56LIN and HPC56POLY. It is also notable that HPC56LIN

584   includes no tropical forcing, although the use of stratospheric polar vortex terms here could

585   incorporate aspects of tropical forcing, such as the ENSO-stratospheric-mid-latitude

586   teleconnection (e.g. Bell et al., 2009). Tropical variables selected are somewhat inconsistent,

587   in terms of both month and predictor, which is likely to be a result of model sensitivity to slight

588   variations due to the short time series used. However, the evidence for tropical signals is clear.

589

590   There are multiple interaction terms identified in the polynomial models, which can be very

591   difficult to explain effectively without subsequent further analysis. N3.4I terms are selected in

592   both 1956 polynomial models, possibly due to the unavailability of MJO indices as discussed

593   above. However, in each case it occurs as an interaction term with a high latitude predictor

594   (terms 9 and 10, stat56POLY; terms 4 and 8, HPC56POLY, Table IVb,c). A closer inspection

595   of the interaction terms' time series reveals that the main input parameter is N3.4I, with the

596   high latitude term modulating the signal in terms of amplitude and sometimes polarity (not

597   shown). This, along with term 1 in stat56POLY (Table IVb), is suggestive of a tropical signal,

598   propagated by Rossby waves, which is then modulated by slowly-varying boundary conditions

599   at higher latitudes (e.g. Ding et al., 2014). Term 5 in HPC56POLY (June Hudson Bay ice x

600   volcanic index; Table IVc) operates in a similar way, the magnitude of the tropical volcanic

601   signal being the dominant input, modulated by the sea-ice term.

602

603   A composite plot of low minus high years for MJO phase 8 in October shows a wave train

604   emanating from the central Pacific, over North America and to the Atlantic (Figure S6).

605   Poleward propagating Rossby waves emanate from only a few source regions, whose activity

606   varies with tropical rainfall variability from year to year (Scaife et al., 2017). Of note in this

607   wavetrain is a node boundary near the Labrador Sea. An interaction term between September

608   Labrador Sea ice and the October MJO8 is identified for stat80POLY (Figure 3, Table IVa). It

609   is possible that in years with high sea-ice in this region, there is an interaction with the MJO

610   signal, and the effect of this interaction is sustained into winter. As the interaction term is

611   multiplicative, then it is the sign of each index which is particularly important. Both high (low)

612   sea-ice and high (low) MJO8 will combine to make the NAO prediction more positive. If,

613   however, one input is positive and the other negative, that will combine towards a more

614   negative NAO forecast.

615

616   **5.3. Higher latitude forcing and interaction terms.**

617   Both cryospheric and extratropical forcings are included in all models, often as interaction

618   terms with tropical forcing as discussed above. Key sea-ice predictors are October Barents-

619   Kara Sea (five out of seven models), November East Siberian -Laptev Sea (four out of seven),

620   Bering Sea (five out of seven). The Greenland Sea has been discussed in Section 5.1 above.

621   Sea-ice in the Barents-Kara Seas has frequently been identified as a key source of potential

622   winter NAO predictability (Scaife et al., 2014; Garcia-Serrano et al., 2015; Wang et al., 2017;

623   Hall et al., 2017). The importance of Bering Sea ice is more surprising, but could well be a

624   proxy for atmospheric variability in the region, such as the Pacific-North American pattern

625   (PNA), with associations with North Atlantic atmospheric variability (e.g. O'Reilly et al.,

626   2017), which in turn is linked to tropical Pacific SST variability.

627

628   Another input variable worthy of more detailed discussion is Barents SLP. This is selected by

629   all NAO models (Figures 3, 4). The most commonly selected month is October (four times).

630   In polynomial models Barents Sea SLP occurs as an interaction term with tropical signals

631   (stat80POLY, Table Iva, HPC56POLY, Table IVc), as a single term (stat56POLY, Table IVb),

632   an interaction term with SSTs (stat56POLY, Table IVb) and the QBO (HPC56POLY, Table

633   IVd). A cyclonic (anticyclonic) anomaly in the BK region in October can lead to positive

634   (negative) sea-ice anomalies there in November (King and Garcia-Serrano, 2016). In other

635   words, the preceding pressure/geopotential height anomaly is a precursor of the Barents-Kara

636   Sea ice ice which has been frequently identified as a predictor in other studies (e.g. Garcia-

637 Serrano et al., 2015; Hall et al., 2017). However, it also has the advantage that it is not subject

638 to the same long-term dramatic trends as autumn sea-ice. The interaction terms support the

639 concept of geopotential height anomalies over the BK Seas in autumn modulating signals from

640 the tropics, stratosphere and cryosphere in a non-linear way (e.g. Vihma et al., in review,

641 International Journal of Climatology). There are a number of input terms from these high

642 latitude regions that are combined in polynomial models: [November east Siberian Sea ice x

643 November GIN seas SST], [October Barents SLP x November GIN seas SST] (stat56POLY);

644 [September GIN seas SST x October Atlantic SST gradient] (HPC56POLY). It seems likely

645 that these terms are capturing different aspects of autumn variability in the northern seas, that

646 would merit further investigation as important predictors of the winter NAO.

647

648 Labrador Sea ice is only selected as a predictor in polynomial models, as an interaction term

649 with tropical (see section 5.2 above) or extratropical forcings. An interaction term selected in

650 both HPC56POLY and stat80POLY is [October Labrador Sea ice x October NAH SST

651 pattern]. The NAH pattern of SSTs is associated with forcing of the winter NAO by persistent

652 SST anomalies at up to six months lead time (Czaja and Frankignoul, 2002). The separate input

653 terms and the resulting multiplicative term are shown in Figure S7a. An examination of the 13-

654 year running correlation between the October NAH and winter NAO reveals a consistent

655 negative correlation, except for the period from 1995 to 2005 when the correlation coefficient

656 increases sharply, becoming positive, before a rapid return to negative values (Figure S7b).

657 This positive excursion coincides with large positive Labrador Sea ice anomalies in October,

658 these being negative for the rest of the time series. The regions for both these variables have a

659 partial overlap. Therefore when there is a negative ice anomaly in the Labrador Sea, the

660 inverted NAH index provides a predictor of the winter NAO interannual variability; however

661 for the brief period with positive ice anomalies, there is a positive relationship between the

662   NAH and NAO interannual variability (Figure S7a). The sea-ice in the Labrador Sea modulates

663   the NAH/NAO interaction, or there is a hidden North Atlantic variable for which Labrador Sea

664   ice is a proxy.

665

666   **6. Summary**

667   The NARMAX approach shows appreciable potential skill in out-of-sample forecasting, albeit

668   with small testing datasets, for both linear and polynomial models which both outperform a

669   more conventional ordinary least squares approach to multiple regression (e.g. Hall et al.,

670   2017). There are strong correlations with observations, reproducing local maxima and minima

671   of the observations, and the amplitude of the observed signal. Model fits are improved when

672   based on a shorter training dataset from 1980-2010. This may partly relate to a wider range of

673   potential predictors being available for this period, but is also because of reduced inherent

674   predictability of circulation indices during the 1950s and 1960s. The skill of polynomial models

675   is consistently greater than that of equivalent linear models, and error statistics are reduced, but

676   small sample size means that further work is needed to confirm the significance of this result.

677   However, an analysis based on using odd years for the testing data is strongly suggestive of a

678   better performance by a polynomial model over that of a linear model, particularly in that it

679   better represents the transition from the early more negative NAO period to the end of the

680   1970s to the more positive phase post-1980. NARMAX can identify important predictors of

681   winter North Atlantic atmospheric variability. Discrepancies between predictor selection in

682   models is likely to arise through increased sensitivity to small fluctuations in input, due to the

683   small sample sizes available. This means that the models capture the same signals, but select

684   them in slightly different ways.

685

686 An important result of the study is that polynomial NARMAX models are capable of revealing

687 the potential modulation of tropical forcing by higher latitude boundary conditions. Barents

688 Sea SLP can also play a crucial role in modulating cryospheric and extratropical signals in

689 addition to those from the tropics. This could be significant in developing the next generation

690 of NWP models.

691

692 Our NARMAX approach can be extended to other circulation indices such as the East Atlantic

693 and Scandinavian Patterns and jet latitude and speed, and to other seasons. This may be

694 especially beneficial for summer seasonal forecasts, where there is currently relatively little

695 predictability from dynamical models (e.g. Ossó et al., 2018; Dunstone et al., 2018). It is also

696 possible to extend the approach to probabilistic forecasting and - by utilising links between

697 North Atlantic circulation patterns and, for example, UK regional temperature and precipitation

698 patterns (Hall and Hanna, 2018) - provide enhanced seasonal forecasts that should be useful

699 for a wide range of stakeholders. NARMAX can also be used to assess how contributions from

700 different atmospheric circulation predictors vary over time using a moving window approach.

701 Future work will use Coupled Model Intercomparison Project (CMIP) 5/6 output to construct

702 models using a longer timeseries, enabling the use of a longer testing dataset, to confirm

703 whether polynomial forecasts are significantly improved compared to linear versions. The

704 models can be further extended to include previous years' predictor values, and by increasing

705 the lead-time at which forecasts are issued for a given season.

706

711

712

713

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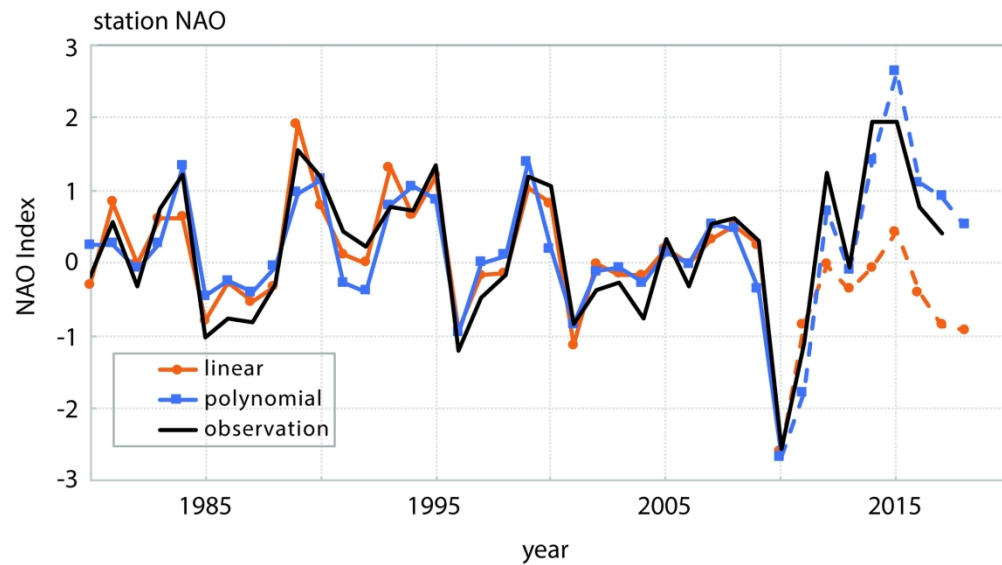
Figure 1 Example of hindcasts (solid) and forecasts (dashed) using linear (LIN) and polynomial (POLY) NARMAX models, derived from the 1980 training period for the station-based NAO.

186x124mm (300 x 300 DPI)
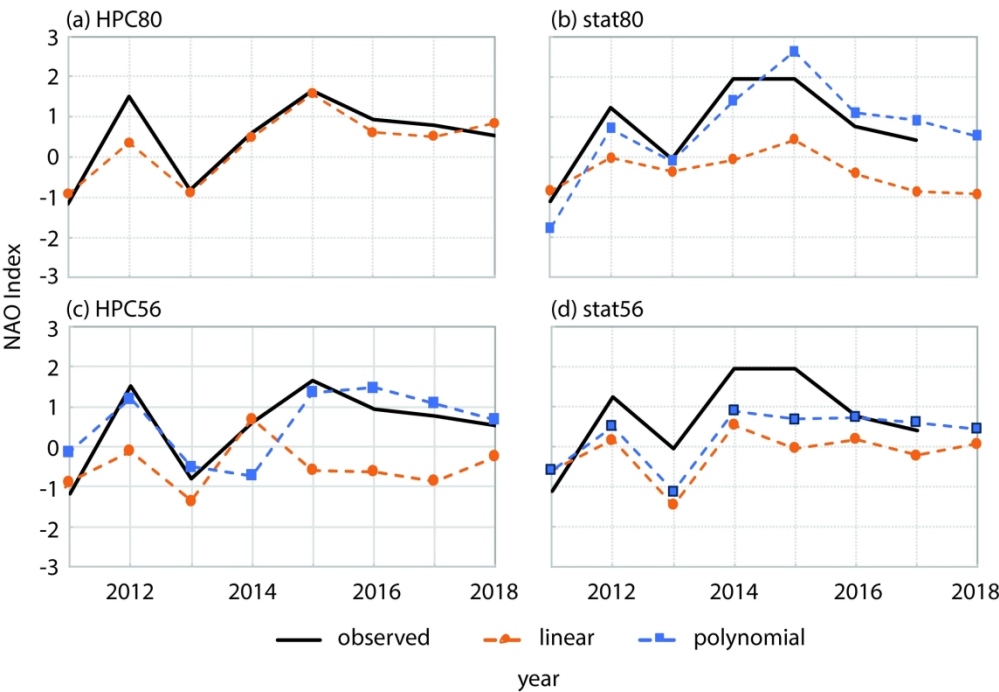
Figure 2. Out-of-sample forecasts compared with observations for linear (LIN) and polynomial (POLY) NARMAX models. Note there is no polynomial model for HPC80 and the observed station NAO is missing for 2018 in (b) and (d).

207x141mm (300 x 300 DPI)
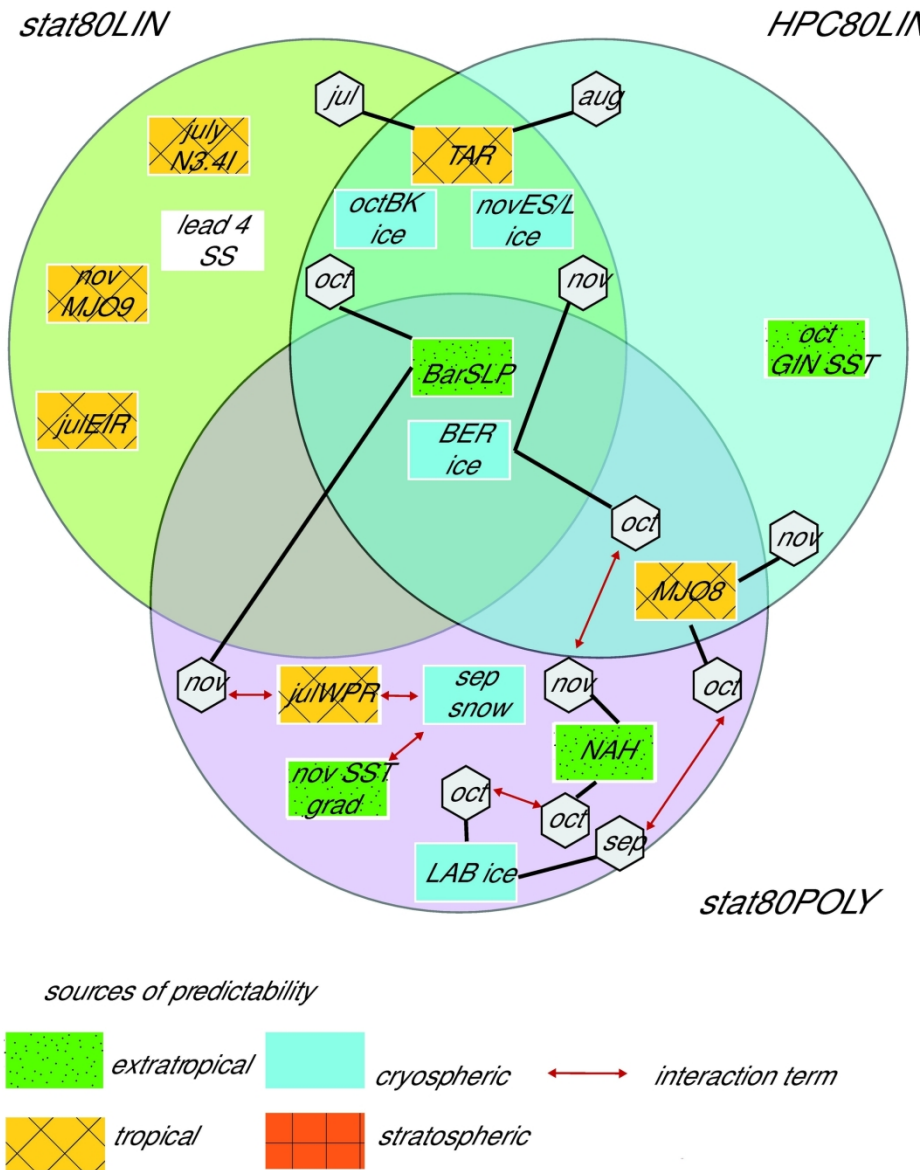
Figure 3. Predictors selected for the NAO models based on the 1980 training period. N3.4I =El Niño 3.4 discontinuous index; EIR=tropical East Indian Ocean rainfall, NAH=North Atlantic Horseshoe SST pattern; snow=Eurasian snowcover; BK ice=Barents-Kara Sea ice; GIN SST= Greenland-Iceland Norwegian SST; ES/L= East Siberian/Laptev Sea ice; TAR= tropical Atlantic rainfall; Bering ice=Bering Sea ice; MJO8/9=Phase 8 Madden-Julian Oscillation; Barents SLP=Barents Sea regional SLP; WPR=tropical West Pacific Ocean rainfall; SST gradient=North Atlantic SST gradient; LAB ice=Labrador Sea ice; lead 4 SS =sunspot cycle leading by 4 years. Where predictor month is not specified, it is indicated by a white hexagon linked to the variable. This is used where different models select different months of a common predictor.

172x225mm (300 x 300 DPI)

Figure 4. As for Figure 3 but using the 1956 training period. Additional variables: volc=volcanic index; SPV=stratospheric polar vortex index; HUD ice=Hudson Bay sea-ice; lead 3 SS=sunspot cycle leading by 3 years; N3.4=standard N3.4 index; CO2=atmospheric carbon dioxide; QBO=Quasi-biennial oscillation; GRE ice =Greenland Sea ice; WISST=tropical West Indian Ocean SST; ARB=Canadian Archipelago/Baffin Bay sea-ice.

219x273mm (300 x 300 DPI)

Table I. Summary of datasets used. For more detailed information, see Table S1.

dataset	Obtained from	Variable used
Hurrell PC_NAO	https://climatedataguide.ucar.edu/climate-data/hurrell-north-atlantic-oscillation-nao-index-pc-based	DJF NAO index
Station-based NAO	https://rda.ucar.edu/datasets/ds570.0	MSLP, Azores and Iceland
AMO	www.climexp.knmi.nl	HadSST3.1.1 SST
HadISST1	www.climexp.knmi.nl	SST, SIC
GPCPv2.3	www.climex.knmi.nl	Tropical precipitation
Carbon dioxide	www.esrl.noaa.gov/gmd/ccgg/trends/data.html	Annual CO_2 level
QBO	www.geo.fu-berlin.de/en/met/ag/strat/produkte/qbo/index.html	Mean zonal wind, 30hPa
sunspots	http://sidc.oma.be	Sunspot number
JRA-55 SPV_T100	Blanca Ayarzaguena, University of Exeter	Temperature 100hPa
NCEP/NCAR SLP	www.climexp.knmi.nl	Sea level pressure
Rutgers Global Snow Lab snow cover extent	https://climate.rutgers.edu/snowcover/docs.php?target=datareq	Snow cover extent
HadCRUT4.6	https://www.metoffice.gov.uk/hadobs/hadcrut4/	2m temperature anomaly
MJO Indices	www.cpc.ncep.noaa.gov/products/precip/CWlink/daily_mjo_index/pentad.html	200hPa velocity potential anomalies

a) stat80LIN	Model Parameter		
Model Term	**7-term**	**8-term**	**9-term**
1 July N3.4I	-0.77	-0.39	-0.23
2 July tropical Atlantic rainfall	-0.10	-0.16	-0.22
3 November Bering Sea ice	0.40	0.46	0.55
4 October Barents-Kara Sea ice	0.53	0.67	0.73
5 October Barents Sea SLP	-0.30	-0.38	-0.37
6 November East Siberian-Laptev Sea ice	-0.32	-0.29	-0.34
7 November MJO phase 9	-0.38	-0.53	-0.63
8 July tropical East Indian Ocean rainfall		0.29	0.30
9 lead 4 year sunspot cycle			-0.18
Individual Model Performance			
Training MAE	0.29	0.27	0.22
data RMSE	0.37	0.31	0.27
correlation	0.92	0.94	0.95
Testing data MAE	1.02	1.25	1.07
RMSE	1.14	1.38	1.26
correlation	0.77	0.83	0.81

b)HPC80LIN	Model Parameter		
Model Term	**7-term**	**8-term**	**9-term**
1 October Barents Kara Sea ice	0.91	0.94	0.85
2 October Barents Sea SLP	-0.42	-0.47	-0.48
3 November Bering Sea ice	0.57	0.35	0.32
4 November East Siberian-Laptev Sea ice	-0.44	-0.47	-0.44
5 constant	0.28	0.27	0.28
6 October GIN SST	0.33	0.42	0.39
7 November MJO phase 8	-0.46	-0.62	-0.57
8 October Bering Sea ice		0.35	0.37
9 August tropical Atlantic rainfall			0.23
Individual Model Performance			
Training MAE	0.33	0.29	0.23
data RMSE	0.41	0.35	0.28
correlation	0.93	0.95	0.97
Testing data MAE	0.33	0.30	0.36
RMSE	0.49	0.47	0.48
correlation	0.92	0.88	0.88

c) stat56LIN	Model Parameter		
Model Term	6-term	7-term	8-term
1 May Greenland Sea ice	-0.15	-0.16	-0.14
2 September NAH	-0.27	-0.27	-0.27
3 July N3.4I	-0.61	-0.65	-0.61
4 October Barents-Kara Sea ice	0.37	0.41	0.44
5 October Barents Sea SLP	-0.23	-0.22	-0.25
6 November tropical West Indian Ocean SST	0.14	0.16	0.14
7 November East Siberian-Laptev Sea ice		-0.16	-0.15
8 October Greenland Sea ice			-0.08
Individual Model Performance			
Training data MAE	0.52	0.51	0.51
RMSE	0.66	0.65	0.64
correlation	0.65	0.67	0.69
Testing data MAE	1.27	0.95	1.03
RMSE	1.38	1.07	1.14
correlation	0.56	0.74	0.73

d) HPC56LIN	Model Parameter		
Model Term	5-term	6-term	7-term
1 October NAH	-0.36	-0.33	-0.28
2 November SPV	-0.38	-0.29	-0.28
3 October SPV	0.41	0.34	0.36
4 July Greenland Sea ice	-0.12	-0.22	-0.27
5 October Barents Sea SLP	-0.31	-0.41	-0.41
6 October Barents-Kara Sea ice		0.44	0.44
7 November Bering Sea ice			0.18
Individual Model Performance			
Training data MAE	0.68	0.61	0.58
RMSE	0.79	0.72	0.69
correlation	0.75	0.80	0.81
Testing data MAE	0.83	1.22	1.29
RMSE	0.99	1.44	1.56
correlation	0.41	0.49	0.42

Table II. Selected predictors, model coefficients and verification statistics for linear NARMAX models.

a) models	MAE		RMSE		correlation		MSESS	D
1980 linear	**training**	**testing**	**training**	**testing**	**training**	**testing**		
stat80LIN	0.25	1.11	0.29	1.26	0.95	0.82	-0.96	0.80
HPC80LIN	0.27	0.29	0.31	0.48	0.96	0.90	0.82	0.75
1980 polynomial								
stat80POLY	0.35	**0.43**	0.41	**0.52**	0.89	**0.92**	**0.68**	**0.89**

b) models	MAE		RMSE		correlation		MSESS	D
1956 linear	**training**	**testing**	**training**	**testing**	**training**	**testing**		
stat56LIN	0.51	1.08	0.64	1.19	0.76	0.69	-0.44	0.80
HPC56LIN	0.61	1.10	0.71	1.31	0.81	0.46	-0.28	0.68
1956 NARMAX								
stat56POLY	**0.40**	**0.68**	**0.50**	**0.81**	**0.87**	**0.77**	**0.34**	0.79
HPC56POLY	**0.40**	**0.59**	**0.52**	**0.70**	**0.90**	**0.73**	**0.68**	**0.79**

Table III. Verification statistics for averaged linear and polynomial NARMAX models, for a) periods 1980-2018 and b) 1956-2018. Bold figures show if polynomial outperforms the linear model.

a)

stat80POLY Model Term	Model Parameter		
	4-term	**5-term**	**6-term**
1 Jul tropical West Pacific Rainfall* Nov Barents Sea SLP	-0.34	-0.31	-0.33
2 Oct Labrador Sea ice LAB * Oct NAH	0.63	0.64	0.58
3 Sep snow * Nov_SST gradient	0.48	0.55	0.56
4 Oct Bering Sea ice * Nov NAH	0.22	0.22	0.27
5 Sep Labrador Sea ice * Oct MJO phase 8		0.51	0.51
6 Jul tropical West Pacific Rainfall * Sep snow			0.33
Individual Model Performance			
Training data MAE	0.40	0.36	0.33
RMSE	0.48	0.43	0.38
correlation	0.84	0.88	0.91
Testing data MAE	0.43	0.53	0.48
RMSE	0.50	0.58	0.62
correlation	0.90	0.88	0.95

b)

stat56POLY Model Term	Parameter		
	8-term	**9-term**	**10-term**
1 Jul tropical West Indian Ocean SST * Sep GIN SST	-0.06	-0.05	-0.07
2 Nov Bering Sea ice	-0.33	-0.67	-0.49
3 Sep NAH	-0.28	-0.28	-0.27
4 Nov East Siberian-Laptev Sea ice * Nov GIN SST	-0.25	-0.19	-0.18
5 Jul Greenland Sea ice * Jul Greenland Sea ice	-0.03	-0.03	-0.03
6 Oct Barents-Kara Sea ice	0.37	0.33	0.32
7 Oct Barents Sea SLP * Nov GIN SST	-0.21	-0.22	-0.24
8 Oct Barents Sea SLP	-0.22	-0.20	-0.21
9 Jul N3.4I * Sep Canadian Archipelago-Baffin sea ice		0.19	0.18
10 Jul N3.4I * Sep NAH			0.46
Individual Model Performance			
Training data RMSE	0.53	0.51	0.49
MAE	0.43	0.40	0.40
Correlation	0.85	0.86	0.87
Testing data RMSE	0.74	0.85	0.85
MAE	0.67	0.68	0.69
Correlation	0.70	0.63	0.82

c)		Parameter		
HPC56POLY	**Model Term**	**6-term**	**7-term**	**8-term**
1 Oct Labrador Sea ice * Oct NAH		0.45	0.41	0.43
2 Aug N3.4 * lead 3 year sunspot cycle		0.63	0.69	0.73
3 atmospheric CO_2 * Jun Greenland Sea ice		0.04	0.04	0.03
4 Nov N3.4I * Sep Barents Sea SLP		0.82	0.97	1.19
5 Jun Hudson Bay sea ice * volcanic index		1.00	1.00	0.99
6 Sep GIN SST * Oct SST gradient		0.25	0.29	0.32
7 Sep QBO * Nov Barents Sea SLP			0.26	0.28
8 Sep N3.4I * Nov GIN SST				-0.56
Individual Model Performance				
Training	RMSE	0.59	0.55	0.49
data	MAE	0.45	0.42	0.37
	Correlation	0.86	0.89	0.91
Testing	RMSE	0.60	0.68	0.75
data	MAE	0.52	0.53	0.57
	Correlation	0.79	0.73	0.68

Table IV. As for Table II, but for polynomial models.

Graphical Abstract

Complex Systems Modelling for Statistical Forecasting of Winter North Atlantic
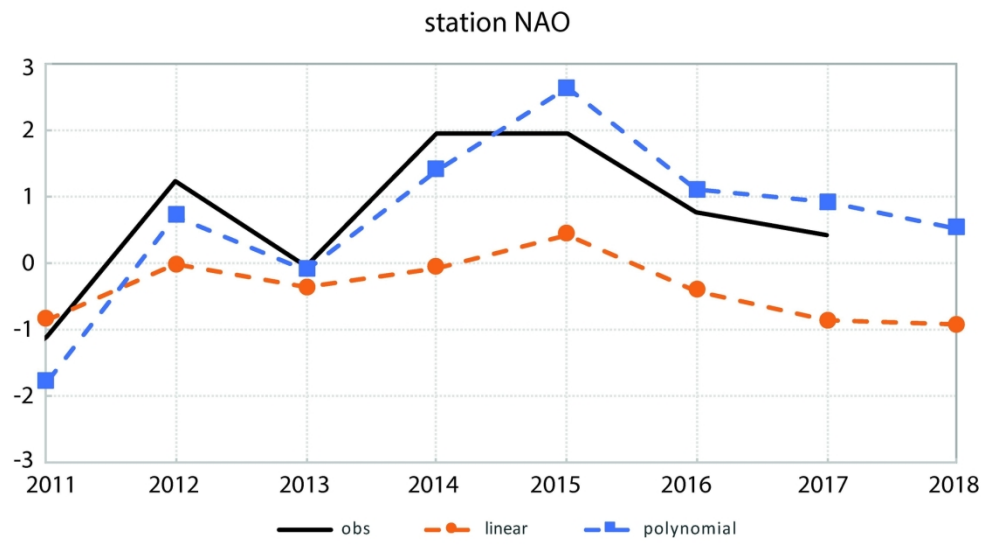
Atmospheric Variability: a New Approach

Richard J. Hall*, Hua-Liang Wei, Edward Hanna

A new approach to seasonal forecasting based on complex systems modelling is presented.

The focus is on North Atlantic winter atmospheric circulation, specifically the NAO.

Polynomial models show greater skill than linear versions and out-of-sample forecasts show

promising skill, closely matching the observed time series. Potential non-linear interactions

between predictors are identified.

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